announcements

- welcome back! 🌞 🌕
- no drill tomorrow
- Problem Set 9 is due on Monday

1

analysis of RM designs

April 10, 2024

2

a hypothetical RM study

- imagine a study where individuals are asked prepare for a quiz using three different strategies: read and reread a passage; answer prepared comprehension questions; create and answer their own comprehension questions
- each person does this once for each strategy (it's a repeated-measures design)
- we counterbalance the order of the strategies
- the outcome is the quiz score (# correct)

3

hypothetical results (matched colors indicate subjects)

student	reread	prepared Qs	create Qs
a	2	5	8
b	3	9	6
c	8	10	12
d	6	13	11
e	5	8	11
f	6	9	12

4

residuals from model w/groups (the usual analysis)

student	reread	prepared Qs	create Qs
a	-3	-4	-2
b	-2	0	-4
c	3	1	2
d	1	4	1
e	0	-1	1
f	1	0	2

residuals are correlated within persons; not good

5

hypothetical results (with marginal means)

student	reread	prepared Qs	create Qs	person *M*s
a	2	5	8	
b	3	9	6	
c	8	10	12	
d	6	13	11	
e	5	8	11	
f	6	9	12	
condition *M*s	**5**	**9**	**10**	

costs 2 parameters to model between-condition differences

6

hypothetical results (with marginal means)

student	reread	prepared Qs	create Qs	person *M*s
a	2	5	8	**5**
b	3	9	6	**6**
c	8	10	12	**10**
d	6	13	11	**10**
e	5	8	11	**8**
f	6	9	12	**9**
condition *M*s	5	9	10	

costs 5 parameters to model between-person differences

7

modeling individual differences with person means

- new Model A

$$\hat{Y} = b_0 + groups + \mathbf{persons}$$

- **this** will cost us *n* – 1 parameters
- but it will gain us power
- and residuals will no longer be correlated within person

8

the model comparison

modeling nonindependence

- Model A: $\hat{Y} = b_0 + groups + persons$
- Model C: $\hat{Y} = b_0 + persons$
- *F*(**2**, 10) = 19.09, *p* = .00038

this is a repeated-measures ANOVA

9

residuals from model w/groups *and* persons as predictors

student	reread	prepared Qs	create Qs
a	0	-1	1
b	0	2	-2
c	1	-2	0
d	-1	2	-1
e	0	-1	1
f	0	-1	1

now residuals are no longer correlated within persons;
and they're lower!

10

but: the RM ANOVA is underinformative

- notice the 2 *df* in the numerator
- this means that two parameters are being clumped together
- it's a better idea to do some *t*-tests!
- these will be *paired-samples* (*related-samples*) t-tests
- be thoughtful about FWER/FDR

11

better than the ANOVA … a series of pairwise comparisons

student	reread	prepared Qs	create Qs
a	2	5	8
b	3	9	6
c	8	10	12
d	6	13	11
e	5	8	11
f	6	9	12

you could do more-complex contrasts if you'd like (e.g., two conditions vs one)

12

more efficient parameterization

13

what *are* parameter estimates?

- imagine a three-condition experiment with the following condition means

  $M_1 = 5$, $M_2 = 9$, $M_3 = 10$
- if we dummy code w/group 1 as the reference
- the parameter estimates will be
  - intercept = 5
  - dummy1 slope = 4
  - dummy2 slope = 5

14

slopes estimate population means & differences among them

- for conditions based on an IV, ***we care*** about these parameter estimates
- but if we estimate additional parameters when trying to manage nonindependence, we get parameters for each person, too
- but ***we don't care*** about the these!
- worse, we're spending one *df* for each person-based parameter that we don't care about

15

modeling individual differences efficiently

- if we care about individual differences and removing them from $MS_{residual}$ (we do) ...
- ... instead of estimating a parameter for each person ...
- ... why not estimate *one parameter to estimate how much everyone differs*?
- this is where variance is useful!

16

using variance to estimate individual differences

- instead of modeling like this

$$\hat{Y} = b_0 + b_1X_1 + b_2X_2 + b_3\text{person} + b_4\text{person} + b_5\text{person} + \cdots$$

- we can model like this

$$\hat{Y} = b_0 + b_1X_1 + b_2X_2 + var(\text{persons})$$

- this will involve estimating a variance between persons, usually called "random intercepts"
- the `lmer` function in the `lme4` package in R makes this easy

17

linear mixed models (LMMs)

- a benefit of modeling RM data w/LMMs is that everything we've learned (dummy variables, interactions, mean-centering, etc.) can be used
- this kind of modeling has become normative in areas of psychology and other fields where nonindependence is common
- in a one-factor RM design with no missing data, the RM ANOVA and its analogous LMM produce identical results
- results no longer converge if the design is more complex or if there are missing data

18

factorial repeated-measures designs

19

design & data

	study time					
	1 minute		2 minutes		3 minutes	
person	abstract	concrete	abstract	concrete	abstract	concrete
a						
b						
c						
d						
e						
mean						

20

design & data

	study time					
	1 minute		2 minutes		3 minutes	
person	abstract	concrete	abstract	concrete	abstract	concrete
a	10	13	12	14	16	17
b	8	12	9	12	11	13
c	12	13	14	14	16	16
d	15	17	16	17	19	20
e	12	13	15	16	16	17
mean						

21

design & data

	study time					
	1 minute		2 minutes		3 minutes	
person	abstract	concrete	abstract	concrete	abstract	concrete
a	10	13	12	14	16	17
b	8	12	9	12	11	13
c	12	13	14	14	16	16
d	15	17	16	17	19	20
e	12	13	15	16	16	17
mean	11.4	13.6	13.2	14.6	15.6	16.6

22

research questions (i.e., contrasts)

- is there an effect of study time?
- is there an effect of word type?
- does the effect of time interact with word type?

23

one way to analyze: contrasts via single-sample *t*-tests

- for each person, find the mean for the abstract condition
- for each person, find the mean for the concrete condition
- subtract the former from the latter
- do a single-sample t-test on the resulting values

	study time							
	1 minute		2 minutes		3 minutes			
person	abstract	concrete	abstract	concrete	abstract	concrete	abstract	concrete
a	10	13	12	14	16	17		
b	8	12	9	12	11	13		
c	12	13	14	14	16	16		
d	15	17	16	17	19	20		
e	12	13	15	16	16	17		
mean	11.4	13.6	13.2	14.6	15.6	16.6		

24

one way to analyze: contrasts via single-sample *t*-tests

- for each person, find the mean for the abstract condition
- for each person, find the mean for the concrete condition
- subtract the former from the latter
- do a single-sample t-test on the resulting values

	study time							
	1 minute		2 minutes		3 minutes			
person	abstract	concrete	abstract	concrete	abstract	concrete	abstract	concrete
a	10	13	12	14	16	17	12.67	
b	8	12	9	12	11	13	9.33	
c	12	13	14	14	16	16	14	
d	15	17	16	17	19	20	16.67	
e	12	13	15	16	16	17	14.33	
mean	11.4	13.6	13.2	14.6	15.6	16.6		

25

one way to analyze: contrasts via single-sample *t*-tests

- for each person, find the mean for the abstract condition
- for each person, find the mean for the concrete condition
- subtract the former from the latter
- do a single-sample t-test on the resulting values

	study time							
	1 minute		2 minutes		3 minutes			
person	abstract	concrete	abstract	concrete	abstract	concrete	abstract	concrete
a	10	13	12	14	16	17	12.67	14.67
b	8	12	9	12	11	13	9.33	12.33
c	12	13	14	14	16	16	14	14.33
d	15	17	16	17	19	20	16.67	18
e	12	13	15	16	16	17	14.33	15.33
mean	11.4	13.6	13.2	14.6	15.6	16.6		

26

one way to analyze: contrasts via single-sample *t*-tests

- for each person, find the mean for the abstract condition
- for each person, find the mean for the concrete condition
- subtract the former from the latter
- do a single-sample t-test on the resulting values

	study time						
	1 minute		2 minutes		3 minutes		
person	abstract	concrete	abstract	concrete	abstract	concrete	d
a	10	13	12	14	16	17	2
b	8	12	9	12	11	13	3
c	12	13	14	14	16	16	0.33
d	15	17	16	17	19	20	1.33
e	12	13	15	16	16	17	1
mean	11.4	13.6	13.2	14.6	15.6	16.6	

27

we could do a subset of simple-effects tests

- within each study time condition, compare abstract vs concrete

	study time					
	1 minute		2 minutes		3 minutes	
person	abstract	concrete	abstract	concrete	abstract	concrete
a	10	13	12	14	16	17
b	8	12	9	12	11	13
c	12	13	14	14	16	16
d	15	17	16	17	19	20
e	12	13	15	16	16	17
mean	11.4	13.6	13.2	14.6	15.6	16.6

28

other options: ezANOVA & all the t-tests

ezANOVA
- pros: easy to set up; conventional
- cons: the omnibus ANOVA is underinformative; focused contrasts difficult (at best) to execute, including "conventional" post-tests

all pairwise *t*-tests
- pros: easy to set up, informative
- cons: scattershot; low power if you care about FWER; may not include all contrasts of interest; no slopes; no SEs; ☹

29

(ez)ANOVA

```
           Effect DFn DFd         SSn   SSd          F            p
1     (Intercept)   1   4 6020.833333 131.0 183.842239 0.0001712670
2       studytime   2   8   65.866667   8.8  29.939394 0.0001929406
3        wordtype   1   4   17.633333   6.2  11.376344 0.0279689588
4 studytime:wordtype 2  8    1.866667   0.8   9.333333 0.0081000000
```

30

all pairwise t-tests

```
         abstract1 abstract2 abstract3 concrete1 concrete2
abstract2 0.1287    -         -         -         -
abstract3 0.0152    0.1389    -         -         -
concrete1 0.2933    1.0000    0.9180    -         -
concrete2 0.0426    0.7741    1.0000    1.0000    -
concrete3 0.0067    0.0717    0.5116    0.0811    0.1658

P value adjustment method: bonferroni
```

31

best option: linear mixed models

- easy to do

```
lmer(dv ~ studytime*wordtype + (1|Subject), twofactorRM)
```

- what does this mean?
- the red part is the usual model
- the blue part is the new thing
- it indicates that we believe that each subject's intercept (i.e., mean) is randomly selected from some population of subject means, and we'd like to know the variance of it

32

LMM output

- ANOVA table

```
                   npar Sum Sq Mean Sq F value
studytime             2 65.867  32.933 41.6878
wordtype              1 17.633  17.633 22.3207
studytime:wordtype    2  1.867   0.933  1.1814
```

- note: *F*-values do not match ezANOVA
- why? it's complicated (different assumptions about what constitutes error/noise, *df* calculation gets ugly)

33