announcements

- happy Monday!
- no drill this week
- Problem Set 9 is due now
- grading is a dream/nightmare

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multifactor RM designs

April 15, 2024

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design & data

	study time					
	1 minute		2 minutes		3 minutes	
person	abstract	concrete	abstract	concrete	abstract	concrete
a	10	13	12	14	16	17
b	8	12	9	12	11	13
c	12	13	14	14	16	16
d	15	17	16	17	19	20
e	12	13	15	16	16	17
mean	11.4	13.6	13.2	14.6	15.6	16.6

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research questions

- is there an effect of study time?
- is there an effect of word type?
- does the effect of time interact with word type?

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one way to analyze: contrasts via single-sample *t*-tests

- for each person, find the mean for the abstract condition
- for each person, find the mean for the concrete condition
- subtract the former from the latter
- do a single-sample t-test on the resulting values

	study time						
	1 minute		2 minutes		3 minutes		
person	abstract	concrete	abstract	concrete	abstract	concrete	d
a	10	13	12	14	16	17	2
b	8	12	9	12	11	13	3
c	12	13	14	14	16	16	0.33
d	15	17	16	17	19	20	1.33
e	12	13	15	16	16	17	1
mean	11.4	13.6	13.2	14.6	15.6	16.6	

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we could do a subset of simple-effects tests

- within each study time condition, compare abstract vs concrete

	study time					
	1 minute		2 minutes		3 minutes	
person	abstract	concrete	abstract	concrete	abstract	concrete
a	10	13	12	14	16	17
b	8	12	9	12	11	13
c	12	13	14	14	16	16
d	15	17	16	17	19	20
e	12	13	15	16	16	17
mean	11.4	13.6	13.2	14.6	15.6	16.6

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other options: ezANOVA & all the t-tests

ezANOVA

- pros: easy to set up; conventional
- cons: the omnibus ANOVA is underinformative; focused contrasts difficult (at best) to execute, including "conventional" post-tests

all pairwise *t*-tests

- pros: easy to set up, informative
- cons: scattershot; low power if you care about FWER; may not include all contrasts of interest; no slopes; no SEs; ☹

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(ez)ANOVA

	Effect	DFn	DFd	SSn	SSd	F	p
1	(Intercept)	1	4	6020.833333	131.0	183.842239	0.0001712670
2	studytime	2	8	65.866667	8.8	29.939394	0.0001929406
3	wordtype	1	4	17.633333	6.2	11.376344	0.0279689588
4	studytime:wordtype	2	8	1.866667	0.8	9.333333	0.0081000000

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all pairwise t-tests

	abstract1	abstract2	abstract3	concrete1	concrete2
abstract2	0.1287	-	-	-	-
abstract3	0.0152	0.1389	-	-	-
concrete1	0.2933	1.0000	0.9180	-	-
concrete2	0.0426	0.7741	1.0000	1.0000	-
concrete3	0.0067	0.0717	0.5116	0.0811	0.1658

P value adjustment method: bonferroni

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best option: linear mixed models

- easy to do

```
lmer(dv ~ studytime*wordtype + (1|Subject), twofactorRM)
```

- what does this mean?
- the red part is the usual model
- the blue part is the new thing
- it indicates that we believe that each subject's intercept (i.e., mean) is randomly selected from some population of subject means, and we'd like to know the variance of it

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LMM output

- ANOVA table

```
                   npar Sum Sq Mean Sq F value
studytime             2 65.867  32.933 41.6878
wordtype              1 17.633  17.633 22.3207
studytime:wordtype    2  1.867   0.933  1.1814
```

- note: *F*-values do not match ezANOVA
- why? it's complicated (different assumptions about what constitutes error/noise, *df* calculation gets ugly)

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why the different *F*-ratios?

- a hint comes from the *df* associated with each effect
- `ezANOVA` (the usual RM ANOVA)

```
$ANOVA
              Effect DFn DFd         SSn   SSd          F
1        (Intercept)   1   4 6020.833333 131.0 183.842239
2          studytime   2   8   65.866667   8.8  29.939394
3           wordtype   1   4   17.633333   6.2  11.376344
4 studytime:wordtype   2   8    1.866667   0.8   9.333333
```

- `lmer` (the LMM) via the `lmerTest` package

```
Type III Analysis of Variance Table with Satterthwaite's method
                   Sum Sq Mean Sq NumDF DenDF F value    Pr(>F)    
studytime          65.867  32.933     2    20 41.6878 7.347e-08 ***
wordtype           17.633  17.633     1    20 22.3207 0.0001298 ***
studytime:wordtype  1.867   0.933     2    20  1.1814 0.3273593    
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

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the *F*-ratio has a different denominator depending on the analysis

- for the RM ANOVA, the denominator for an effect is the interaction of the effect with participants?
- what?!
- let's look at the data again

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a	10	13	12	14	16	17	**2**
b	8	12	9	12	11	13	**3**
c	12	13	14	14	16	16	**0.33**
d	15	17	16	17	19	20	**1.33**
e	12	13	15	16	16	17	**1**
mean	11.4	13.6	13.2	14.6	15.6	16.6	

	study time							
	1 minute		2 minutes		3 minutes			
person	abstract	concrete	abstract	concrete	abstract	concrete	abstract	concrete
a	10	13	12	14	16	17	**12.67**	**14.67**
b	8	12	9	12	11	13	**9.33**	**12.33**
c	12	13	14	14	16	16	**14**	**14.33**
d	15	17	16	17	19	20	**16.67**	**18**
e	12	13	15	16	16	17	**14.33**	**15.33**
mean	11.4	13.6	13.2	14.6	15.6	16.6		

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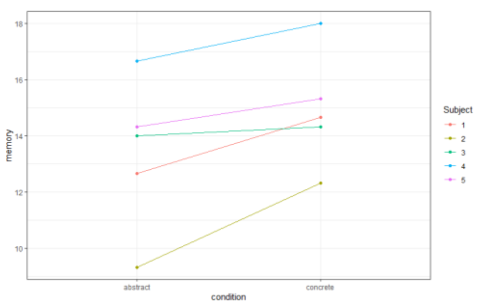

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d	15	17	16	17	19	20	**1.33**
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- Model C: $\hat{d} = 0$ (no parameters)
- Model A: $\hat{d} = b_0$ (one parameter)
- Model C SSE = 15.889
- Model A SSE = 4.133

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c	12	13	14	14	16	16	**0.33**
d	15	17	16	17	19	20	**1.33**
e	12	13	15	16	16	17	**1**
mean	11.4	13.6	13.2	14.6	15.6	16.6	

$$F = \frac{SSR/df}{SSE(A)/df}$$

$$F = \frac{SSR/df}{\mathbf{4.133}/df}$$

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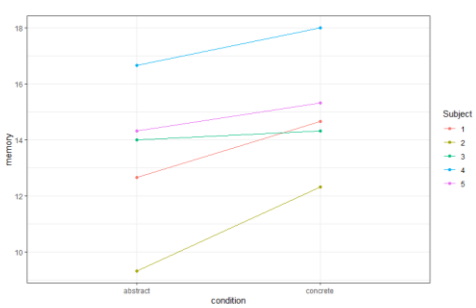

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the *F*-ratio has a different denominator depending on the analysis 🥴

- for the LMM, the denominator for all effects is the same: it's the SS for the residuals

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more multilevel modeling

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What is this about?

- Imagine we are interested in the extent to which a pre-test (X; mean-centered!) predicts standardized math test scores (Y) in 5th graders.
- We collect data from one classroom and find:

$$\hat{Y}_i = \beta_0 + \beta_1 X_i$$
$$\hat{Y}_i = 70 + 0.2X_i$$

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A complication

- Imagine that we collected more data for a second classroom and found this:

$$\hat{Y}_i = 60 + 0.2X_i$$

- Different intercept (maybe the class has a different overall level of ability)

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What should we do?

- Three options, from least to most complex:
  1) Combine the data across classes and ignore that they come from different classes
  2) Acknowledge that the data come from different classes and include classrooms as a part of our regression model
  3) Multilevel modeling

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Option 1

- Collapsing across classes
- This gives us:

$$\hat{Y}_i = 65 + 0.2X_i$$

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Option 2

- Modeling the classroom, too
- Using a dummy-code (classroom 1 = 0)
- This gives us

$$\hat{Y}_i = \beta_0 + \beta_1 X_{pretest,i} + \beta_2 X_{class\ i}$$
$$\hat{Y}_i = 70 + 0.2 X_{pretest,i} + (-10) X_{class,i}$$

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Option 3

- Modeling not only the effect of the pretest at the subject level
- Also modeling the differences in classrooms

$$\hat{Y}_i = \hat{\beta}_{0,j} + \beta_1 X_{pretest,i} + e_i$$
$$\hat{\beta}_{0,j} = \gamma_0 + u_j$$

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Option 3

- Modeling not only the effect of the pretest at the subject level
- Also modeling the differences in classrooms

$$\hat{Y}_i = \hat{\beta}_{0,j} + \beta_1 X_{pretest,i} + e_i$$
$$\hat{\beta}_{0,j} = \gamma_0 + u_j$$

- This is called a random-intercept model, and can be presented as one equation

$$\hat{Y}_i = [\gamma_0 + \beta_1 X_{pretest,i}] + [u_j + e_i]$$

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