categorical predictors
(the end, sort of)

February 14, 2024

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writing about results

Three pairwise comparisons were executed by orthogonal contrasts. To control the Type I error rate, a Bonferroni-corrected α = .05/3 = .017 was used. The imagery group (*M* = 12) had significantly better memory than the control group (*M* = 6), *t*(27) = 3.48, *p* = .001. The rhyme group (*M* = 10) had non-significantly better memory than the control group (*M* = 6), *t*(27) = 2.32, *p* = .03. The imagery and rhyme groups also did not differ significantly, *t*(27) = 1.16, *p* = .26.

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or …

Three pairwise comparisons were executed by orthogonal contrasts. To control the Type I error rate, Bonferroni-corrected *p*-values were used with α = .05. The imagery group (*M* = 12) had significantly better memory than the control group (*M* = 6), *t*(27) = 3.48, *p* = .005. The rhyme group (*M* = 10) had non-significantly better memory than the control group (*M* = 6), *t*(27) = 2.32, *p* = .085. The imagery and rhyme groups also did not differ significantly, *t*(27) = 1.16, *p* = .77.

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a little theory

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reminders about SSE and SSR

- in a design with three groups, Model A is

$$Y_{ij} = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \varepsilon_{ij}$$

- we can express predicted scores as follows

$$\hat{Y}_{ij} = b_0 + b_1 X_1 + b_2 X_2 \quad \text{or} \quad \hat{Y}_{ij} = \bar{Y}_{.j}$$

- and we can express (estimates of) residuals

$$e_{ij} = Y_{ij} - \bar{Y}_{.j}$$

- and SSE(A) is

$$SSE(A) = \sum (Y_{ij} - \bar{Y}_{.j})^2$$

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reminders about SSE and SSR

- in a design with three groups, Model C (for the usual ANOVA) is

$$Y_{ij} = \beta_0 + 0X_1 + 0X_2 + \varepsilon_{ij}$$

- we can express predicted scores as follows

$$\hat{Y}_{ij} = b_0 \quad \text{or} \quad \hat{Y}_{ij} = \bar{Y}_{..}$$

- and we can express (estimates of) residuals

$$e_{ij} = Y_{ij} - \bar{Y}_{..}$$

- and SSE(C) thus is

$$SSE(C) = \sum \left(Y_{ij} - \bar{Y}_{..}\right)^2$$

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reminders about SSE and SSR

- if we compare Model A to Model C, we get SSR

$$SSR = SSE(C) - SSE(A)$$

- SSR is the reduction (improvement) in SSE
- it can be re-expressed as follows

$$SSR = \sum n_j(\bar{Y}_{.j} - \bar{Y}_{..})^2$$

- or (less formally, but more clearly, I hope)

$$SSR = \sum n_{\text{group}}(\bar{Y}_{\text{group}} - \bar{Y}_{\text{overall}})^2$$

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these SS values have aliases in the context of ANOVA

$$SSR = SS_{\text{between}}$$
$$SSE(A) = SS_{\text{within}}$$
$$SSE(C) = SS_{\text{total}}$$

- SS_{between} is a measure of differences between groups, with sample size playing a role
- Why do group means differ?
  - real differences + noise
- SS_{within} is a measure of differences within groups
- Why do scores within groups differ?
  - noise

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now MSs

- the *df* associated with *SS*s can be used to calculate *MS* values, as follows

$$MS_{\text{between}} = {SS_{\text{between}}}/{k-1}$$

$$MS_{\text{within}} = {SS_{\text{within}}}/{n-k}$$

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finally the *F*-ratio

$$F = \frac{MS_{\text{between}}}{MS_{\text{within}}}$$

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what contributes to the *F*-ratio?

formally

$$F = \frac{E(MS_{\text{between}})}{E(MS_{\text{within}})} = \frac{\sigma_{\text{e}}^2 + n\theta_{\text{groups}}^2}{\sigma_{\text{e}}^2}$$

what?!

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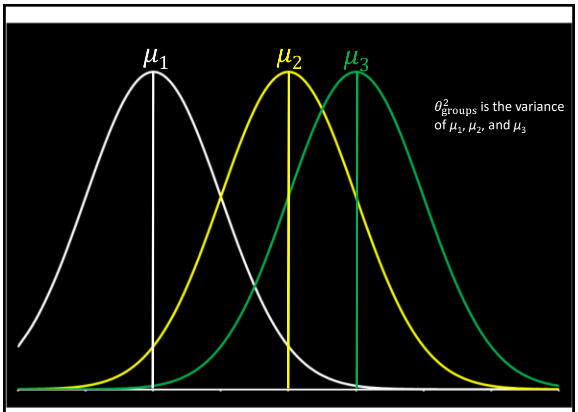

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what contributes to the *F*-ratio?

formally

$$F = \frac{E(MS_{\text{between}})}{E(MS_{\text{within}})} = \frac{\sigma_e^2 + n\theta_{\text{groups}}^2}{\sigma_e^2}$$

informally!

$$F = \frac{\text{noise} + \text{sample size} \times \text{group diffs}}{\text{noise}}$$

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why does this matter?

$$F = \frac{\text{noise} + \text{sample size} \times \text{group diffs}}{\text{noise}}$$

- if noise is minimized, power goes up
- if sample size is increased, power goes up
- if groups are more different, power goes up

- this also is how an *F*-ratio is constructed: if there are no group diffs (it's 0), the numerator and denominator are both noise and *F* is expected to equal 1

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power analysis

- for three or more groups, you have choices to make
- one (sub-optimal) choice is to power the ANOVA
  - you'll probably use f^2 to do this
- better is to power particular contrasts, whether they be pairwise comparisons of more complex comparisons
  - you can use f^2 here as well
  - for pairwise comparisons, you can use Cohen's *d* (in G*Power, at least)
- effect sizes can be converted (see here)
- there are lots of resources and packages (e.g., Superpower)

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recommendations

- read widely in your field to know what is standard *now*
  - are ANOVAs still being done with post-tests?
  - are models being fit with some sort of coding?
- if you do not create your own predictors, make sure you know what your software is using

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