welcome again

January 22, 2024

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polynomial/power predictors
("nonlinear" regression)

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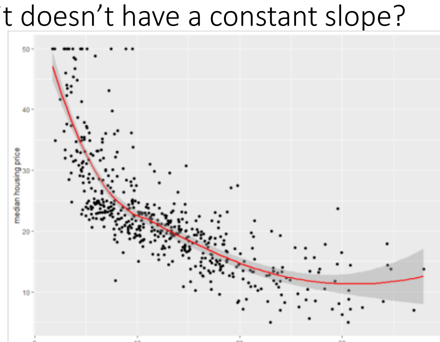
what if there's only one predictor but it doesn't have a constant slope?



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some terminology

- we can handle this nonlinearity with a linear model
- how can a nonlinear relationship be handled by a linear model?
- when a predictor is transformed (e.g., by squaring), the relationship between the transformed predictor and the outcome is linear

$$Y = b_0 + b_1X + b_2X^2$$

- as X^2 increases, Y increases linearly

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a sample data set

- outcome = time to complete 5K race
- predictor = number of miles run per week (training)

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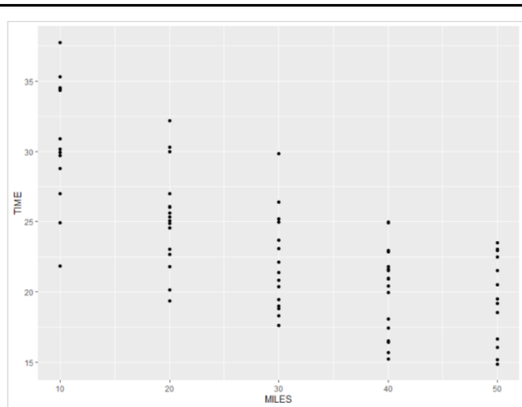

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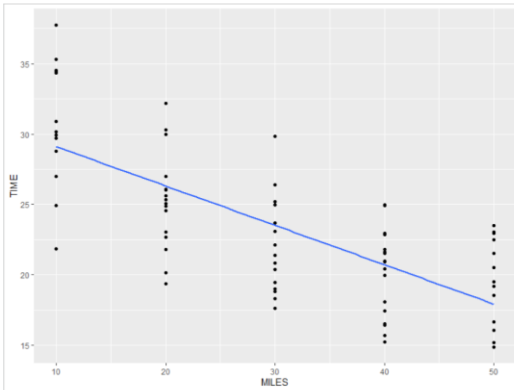

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fitting a simple model

$$\widehat{TIME} = b_0 + b_1 MILES$$

- b_1 = -0.28, SE = 0.03, $t(78)$ = -9.47, p = very small
- this suggests that more training is associated with shorter race times

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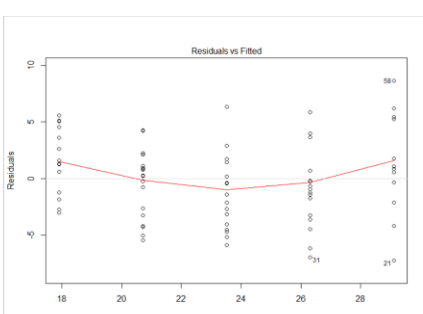
what’s wrong with this model?



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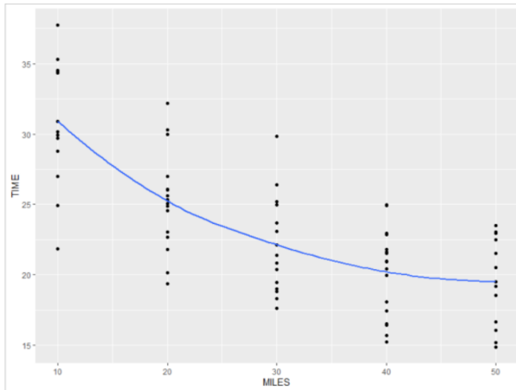

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we can add a parameter to model the change in slope

$$\widehat{TIME} = b_0 + b_1 MILES$$

$$\widehat{TIME} = b_0 + b_1 M + b_2 M^2$$

- b_1 captures the "linear" relationship **at MILES = 0**
- $2b_2$ captures the change in b_1 as miles changes
- why **2**b_2? because the slope of a tangent line is based on the derivative of a function
- the derivative of M^2 is $2M$

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interpreting parameters

```
             Estimate Std. Error t value
(Intercept) 22.053552   0.581916  37.898
MILES.c     -0.279100   0.027734 -10.063
M2           0.007941   0.002331   3.407
```

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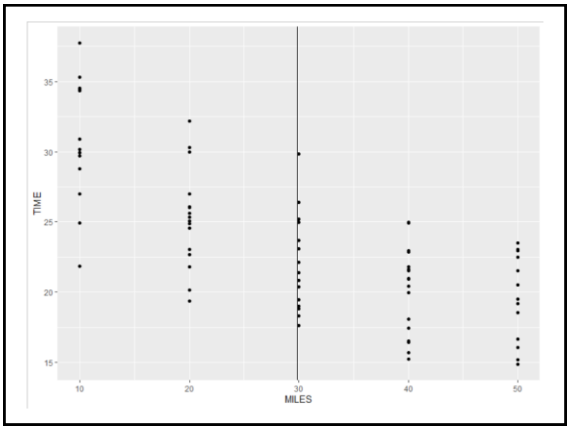
TIME
MILES
35
30
25
20
15
10
20
30
40
50

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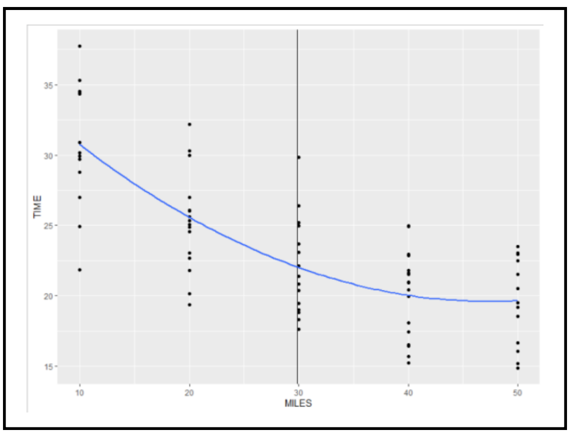
TIME
MILES
35
30
25
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40
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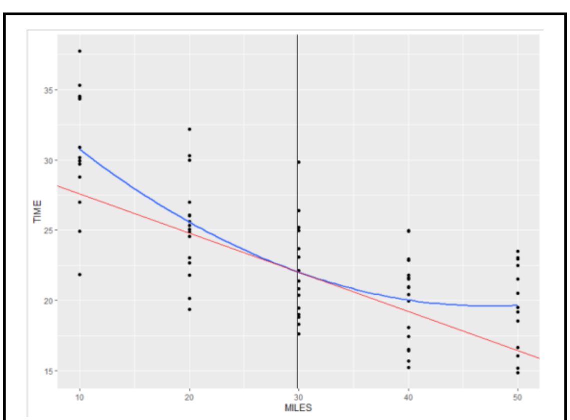
TIME
MILES
35
30
25
20
15
10
20
30
40
50

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what would the slope be at miles = 40?

- the long way

$$\text{TIME} = 22.05 - 0.28M_c + 0.008M_c^2$$

$$\text{slope} = -0.28 + 2*0.008M_c$$

- in M_c terms, 40 miles is 10.125

$$\text{slope} = -0.28 + 2*0.008(10.125)$$

$$\text{slope} = -0.12$$

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what would the slope be at miles = 40?

- the easy way
- center miles at 40 and re-do the analysis

```
 (Intercept)          M40         M40sq
20.041722950 -0.118298736  0.007940781
```

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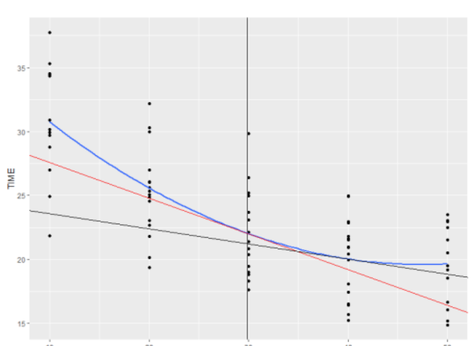

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